

Answers

PPY#2

Find a) $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ b) evaluate both at $t=0$

$$1. \quad x = t^2 + 3t \quad y = t + 1$$

$$\begin{aligned} \textcircled{1} \text{ a) } \frac{dy}{dx} &= \frac{1}{2t+3} \\ -\frac{d^2y}{dx^2} &= \frac{-2}{(2t+3)^3} \end{aligned}$$

$$\text{b) } \frac{1}{3} - \frac{2}{27}$$

Write the eqn of the tangent line to the curve at the given point.

$$2. \quad x = 2t \quad y = t^2 \quad \text{at } t=2$$

$$\textcircled{2} \quad y - 3 = 2(x - 4)$$

$$3. \quad x = t^2 - t + 2 \quad y = t^3 - 3t \quad \text{at } t = -1$$

$$\textcircled{3} \quad y = 2$$

Find all pts (if any) of HT and VT to the curve.

$$4. \quad x = 1 - t \quad y = t^2$$

$$\textcircled{4} \quad \text{HT } (1, 0) \quad \text{VT: none}$$

$$5. \quad x = 3\cos\theta \quad y = 3\sin\theta$$

$$\textcircled{5} \quad \text{HT } (0, 3) (0, -3) \quad \text{VT } (3, 0) (-3, 0)$$

Find the arclength (use fint)

$$6. \quad x = \arcsint \quad y = \ln\sqrt{1-t^2} \quad 0 \leq t \leq \frac{1}{2}$$

$$\textcircled{6} \quad \int_0^{y_2} \sqrt{\frac{1}{(1-t^2)^2}} dt \approx 0.549$$

Find $\frac{dy}{dx}$ and evaluate it at $\theta = \frac{\pi}{2}$

$$7. \quad r = 3(1 - \cos\theta)$$

$$\textcircled{7} \quad \frac{dy}{dx} = \frac{(1+2\cos\theta)(1-\cos\theta)}{\sin\theta(2\cos\theta-1)} \quad @ \theta = \frac{\pi}{2} \quad \frac{dy}{dx} = -1$$

Find the area of the given region:

8. One leaf of $r = 2\cos 3\theta$

$$\textcircled{8} \quad 2 \left[\frac{1}{2} \int_0^{\frac{\pi}{6}} (2\cos 3\theta)^2 d\theta \right] = \frac{\pi}{3}$$

9. Inner loop of $r = 1 + 2\cos\theta$

$$\textcircled{9} \quad 2 \left[\frac{1}{2} \int_{\frac{2\pi}{3}}^{\pi} (1+2\cos\theta)^2 d\theta \right] = \frac{2\pi - 3\sqrt{3}}{2}$$

10. Between the loops of $r = 1 + 2\cos\theta$

$$\textcircled{10} \quad 2 \left[\frac{1}{2} \int_0^{\frac{2\pi}{3}} (1+2\cos\theta)^2 d\theta \right] = \frac{4\pi + 3\sqrt{3}}{2} \quad \begin{matrix} \text{outer loop} \\ \text{between loops answer to } 10 - 9 = \pi + 3\sqrt{3} \\ \text{outer-inner} \end{matrix}$$

$$\textcircled{11} \quad (1, \frac{\pi}{2}), (1, \frac{3\pi}{2}), (0, 0)$$

$$\textcircled{12} \quad 2 \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \sqrt{(1+\sin\theta)^2 + (\cos\theta)^2} d\theta = 8$$

Find the intersection points of

$$11. \quad r = 1 + \cos\theta \quad \text{and} \quad r = 1 - \cos\theta$$

Find the arclength

$$12. \quad r = 1 + \sin\theta \quad 0 \leq \theta \leq 2\pi$$

Find the a) position at time $t=3$, b) distance the particle travels $t \in [0, 3]$ use fint

$$13. \quad v(t) = \langle 3t^2 - 2t, 1 + \cos\pi t \rangle \quad s(0) = (2, 6)$$

$$\textcircled{13} \text{ a) } \langle 20, 9 \rangle$$

$$\text{a) } \left\langle \int_0^3 3t^2 - 2t dt, \int_0^3 1 + \cos\pi t dt \right\rangle + \langle 2, 6 \rangle$$

$$\text{b) } \int_0^3 \sqrt{(3t^2 - 2t)^2 + (1 + \cos\pi t)^2} dt \approx 19.343 \text{ (fint)}$$

